

Midterm 1 – Review – Problems

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1 Epsilon-Delta

Problem 1

Use an ϵ - δ argument to show:

$$\lim_{x \rightarrow 5} 2x + 3 = 13$$

2 Limits

Problem 2

Find the following limits

(a) $\lim_{x \rightarrow 3} \frac{x^2 - 5x + 6}{x^2 - 9}$

(b) $\lim_{x \rightarrow 2} \frac{x^2 - x - 6}{x - 2}$

(b) $\lim_{x \rightarrow 3^-} \frac{\ln(x)}{x - 3}$

(c) $\lim_{x \rightarrow 0^+} \frac{(\ln(x))^2 + 1}{(\ln(x))^2 + 3}$

(d) $\lim_{x \rightarrow 0} \sin\left(\frac{\pi|x|}{x}\right)$

(e) $\lim_{x \rightarrow 3} \frac{x - 3}{\sqrt{x} - \sqrt{3}}$

(g) $\lim_{x \rightarrow 0} \frac{\cot(2x)}{x}$

(h) $\lim_{x \rightarrow -\infty} \frac{\sqrt{x^4 + 1}}{x^2}$

3 Continuity and the IVT

Problem 3

Is the following function f continuous at 0?

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 0 \\ x \sin\left(\frac{1}{x}\right) & \text{if } x > 0 \end{cases}$$

Problem 4

Show that $x^4 - x = 3$ has at least one solution

4 Differentiability

Problem 5

Is the following function f differentiable at $x = 1$?

$$f(x) = \begin{cases} 2x - 1 & \text{if } x < 1 \\ x^2 & \text{if } x \geq 1 \end{cases}$$

Problem 6

Is the following function f differentiable at $x = 0$?

$$f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

Problem 7

Give an example of a function which is continuous at 0, but not differentiable at 0.

5 Derivatives

Problem 8

Find the derivatives of the following functions, using the **definition** of the derivative:

(a) $f(x) = \frac{1}{x^2}$

(b) $f(x) = \sqrt{1 + 3x}$

Problem 9

Show that there is no tangent line to the curve $y = x^2$ that goes through $(2, 16)$

6 Inverse-Trig Stuff**Problem 10**

Let $f(x) = \cos^{-1}(e^x)$

- (a) Find the domain of f
- (b) Find the domain of f^{-1}
- (c) Show that f is one-to-one
- (d) Find a formula for $f^{-1}(x)$